Warm-up!

Find the next number in the pattern

A) $3, 8, 13, 18, 23, \ldots$ (+5 +5 +5 +5) arithmetic

B) $4, 8, 16, 32, 64, \ldots$ (×2 ×2 ×2 ×2) ← geometric

C) $27, -18, 12, -8, \frac{16}{3}, \ldots$ ($\times -\frac{2}{3}$ $\times -\frac{2}{3}$)

$\frac{-18}{27} = -\frac{2}{3}$ $\quad \frac{12}{-18} = -\frac{2}{3}$

D) $1, -3, 5, -7, 9, \ldots$

Exponential Functions Day 1:

- (2.1) Arithmetic Sequences
- (2.1) Geometric Sequences

Homework

- Intro to Arithmetic and Geometric Sequences

Your next quiz will be Day 5 of this unit

Sequence

Definition: A function whose domain is the set of nonnegative integers

Examples:

- $2, 4, 6, 8$
- $200, 100, 50, 25, \ldots$
- Monday, Tuesday, Wednesday,...

Sequence Terms: Instead of representing sequences with function notation, we use subscripts.

Instead of $f(1) = 2, f(2) = 4$, and $f(x) = 2x$,

We write

$a_1 = 2$

$a_2 = 4$

$a_n = 2n$

(we often use the letter $a$, but sometimes we'll use a different letter for reasons that will be clear later)

Given the general term for a sequence, find the first four terms in the sequence.

$f(n) = 12 - 5n$

A) $a_n = 12 - 5n$

$a_1 = 12 - 5(1) = 7$

$a_2 = 12 - 5(2) = 2$

$a_3 = 12 - 5(3) = -3$

$a_4 = 12 - 5(4) = -8$

$7, 2, -3, -8, -13, \ldots$

B) $a_n = 32\left(-\frac{1}{2}\right)^n$

$a_1 = 32\left(-\frac{1}{2}\right)^1 = -16$

$a_2 = 32\left(-\frac{1}{2}\right)^2 = 8$

$a_3 = 32\left(-\frac{1}{2}\right)^3 = -4$

$a_4 = 32\left(-\frac{1}{2}\right)^4 = 2$

$-16, 8, -4, 2$

C) $a_n = \frac{n-1}{n+1}$

$a_1 = \frac{0}{2}$ $\quad a_2 = \frac{1}{3}$ $\quad a_3 = \frac{2}{4}$ $\quad a_4 = \frac{3}{5}$

$0, \frac{1}{3}, \frac{1}{2}, \frac{3}{5}$

$a_1, a_2, a_3, a_4$

Some types of sequences get a special name

From the warm-up:

A) $3, 8, 13, 18, \ldots$

B) $4, 8, 16, 32, \ldots$

C) $27, -18, 12, -8, \ldots$

D) $1, -3, 5, -7, \ldots$

Geometric Sequence:

- Always MULTIPLY by the same amount to get from one term to the next
- Amount you multiply by = "Common Ratio" = r
- If you're ever wondering what the common ration is, pick a random term and divide by the term before it.

Arithmetic Sequence

- Always ADD the same amount to get from one term to the next
- Amount you add = "Common Difference" = d
- If you're ever wondering what the common difference is, pick a random term and subtract the term before it.

A) $3, 8, 13, 18, \ldots$

$d = 5$

B) $4, 8, 16, 32, \ldots$

$R = 2$

C) $27, -18, 12, -8, \ldots$

$R = -\frac{2}{3}$

D) $1, -3, 5, -7, \ldots$

Find the first 4 terms in the sequence. Classify the sequence as arithmetic, geometric, or neither. If it is arithmetic, find the common difference. If geometric, find the common ratio.

D) $a_n = 3 - 7n$

$-4, -11, -18, -25, \ldots$

arithmetic $d = -7$

E) $a_n = 3 \cdot (-2)^n$

$a_1$ $a_2$ $a_3$ $a_4$

$-6, 12, -24, 48, \ldots$

geometric $R = -2$

F) Find the eighth term in the following arithmetic sequence

$a_1 = 4$ +6
$a_2 = 10$ +6
$a_3 = 16$
…

$$a_8 = a_3 + 5d$$

$$a_8 = 16 + 5(6)$$

$$16 + 30$$

$$a_8 = 46$$

G) Find the eightieth term in the following arithmetic sequence

$a_0 = -2$

$a_1 = 4$

$a_2 = 10$

$a_3 = 16$

...

$a_{80} = a_8 + 72d$

$a_{80} = a_0 + 80d$

$-2 + 80(6)$

$a_{80} = a_3 + 77d$

$a_{80} = 16 + 77(6)$

$a_{80} = 16 + 462$

$a_{80} = 478$

When you know a term and the common difference/ratio and want to come up with another term or the general formula

## Finding the nth term of an arithmetic sequence:

$$a_n = a_k + d(n-k)$$

where

$a_k$ = term you know
$d$ = common difference
$a_n$ = n-th term

So,

$a_n = a_1 + d(n-1)$
$a_n = a_2 + d(n-2)$
$a_n = a_3 + d(n-3)$
$a_n = a_4 + d(n-4)$
$a_n = a_5 + d(n-5)$
…

## Finding the nth term of a geometric sequence:

$$g_n = g_k(r)^{n-k}$$

where

$g_k$ = term you know
$r$ = common ratio
$g_n$ = n-th term

So,

$g_n = g_1 \cdot r^{n-1}$
$g_n = g_2 \cdot r^{n-2}$
$g_n = g_3 \cdot r^{n-3}$
$g_n = g_4 \cdot r^{n-4}$
$g_n = g_5 \cdot r^{n-5}$
…

Given a term and a common ratio/difference, finding another term

H) Find the 20th term of the arithmetic sequence with $a_3 = 7$ and $d = 4$

$a_{20} = a_3 + 17d$

$a_{20} = 7 + 17(4) = 7 + 68 = 75$

$a_{20} = 75$

I) Find the 15th term of the geometric sequence with $g_6 = 20$ and $r = \frac{1}{3}$. You may leave your answer unsimplified.

$g_{15} = g_6 \cdot R^9$

$g_{15} = 20\left(\frac{1}{3}\right)^9 = \frac{20}{3^9}$

$2^0$
$2^1$
$2^2$
$2^3$
$2^4$
$2^5$
$2^6$

$3^0$
$3^1$
$3^2$
$3^3$

$4^0$
$4^1$
$4^2$
$4^3$

$5^0$
$5^1$
$5^2$

Given a term and a common ratio/difference, finding another term

J) Find the 85th term of the arithmetic sequence with $a_3 = 300$ and $d = -2$

$a_{85} = a_3 + 82d$

$a_{85} = 300 + 82(-2) = 300 - 164$

$= 136$

K) Find the $nth$ term of the geometric sequence with $g_4 = 10$ and $r = 3$.

$g_n = g_4(3)^{n-4}$

$g_n = 10(3)^{n-4}$

$g_4 = 10$

$g_5 = 10R$

$g_6 = 10R^2$

$g_7 = 10R^3$

$g_n = 10R^{n-4}$

Finding General Formulas

L) Find a formula for the $n$th term of the arithmetic sequence with $a_5 = 15$ and $d = 4$.

$a_n = a_5 + 4(n-5)$

$a_n = 15 + 4(n-5)$

$a_n = 4n - 5$

M) Find a formula for the $n$th term of the geometric sequence with $a_3 = 10$ and $r = -\frac{2}{3}$

$a_n = a_3 R^{n-3}$

$a_n = 10\left(-\frac{2}{3}\right)^{n-3}$

$$\frac{10\left(-\frac{2}{3}\right)^{n}}{\left(-\frac{2}{3}\right)^{3}}$$

Finding the common difference or ratio when given only two terms

For Arithmetic Sequences

- Subtract the terms
- Divide by the difference in the indices
- (make sure you subtract in the right order)

N) Find the common difference of an arithmetic sequence with $a_1 = -4$ and $a_5 = 16$

$a_5 = a_1 + 4d$

$16 = -4 + 4d$

$20 = 4d$

$5 = d$

For Geometric Sequences

- Divide the terms
- Root by the difference in the indices

O) Find the common ratio of a geometric sequence with $g_2 = 56$ and $g_5 = 7$

$g_5 = g_2 R^3$

$7 = 56 R^3$

$\frac{7}{56} = R^3$

$\frac{1}{8} = R^3$

$\frac{1}{2} = R$

P) Given the following terms from an arithmetic sequence, find the common difference. Then, find a general formula for the $n$th term in the sequence.

$a_5 = 190$ and $a_{10} = 115$

$a_{10} = a_5 + 5d$

$115 = 190 + 5d$

$-75 = 5d$

$-15 = d$

$a_n = a_5 + (n-5)d$

$a_n = 190 + (-15)(n-5)$

$a_n = 190 - 15(n-5)$

$a_n = 190 - 15n + 75$

$a_n = 265 - 15n$

$(5, 190)$

$(10, 115)$

$\frac{115 - 190}{10 - 5}$

$\frac{-75}{5}$

$-15$

$a_n = a_{10} + (n-10)d$

$a_n = 115 + (n-10)(-15)$

$a_n = 115 - 15n + 150$

$a_n = 265 - 15n$

Q) Given the following terms from a geometric sequence, find the common ratio. Then, find a general formula for the $n$th term in the sequence.

$g_{10} = -5$ and $g_{15} = 160$

$g_{15} = g_{10} R^5$

$160 = -5 R^5$

$\sqrt[5]{-32} = \sqrt[5]{R^5}$

$-2 = R$

$g_n = g_{10} (-2)^{n-10}$

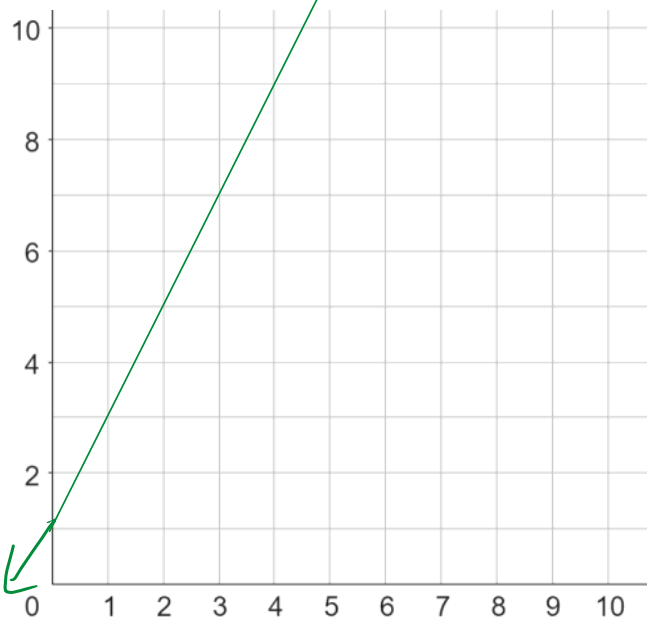
$g_n = -5 (-2)^{n-10}$

$g_n = g_{15} (-2)^{n-15}$

$g_n = 160 (-2)^{n-15}$

Graphing Sequences

$f(x) = 2x + 1$

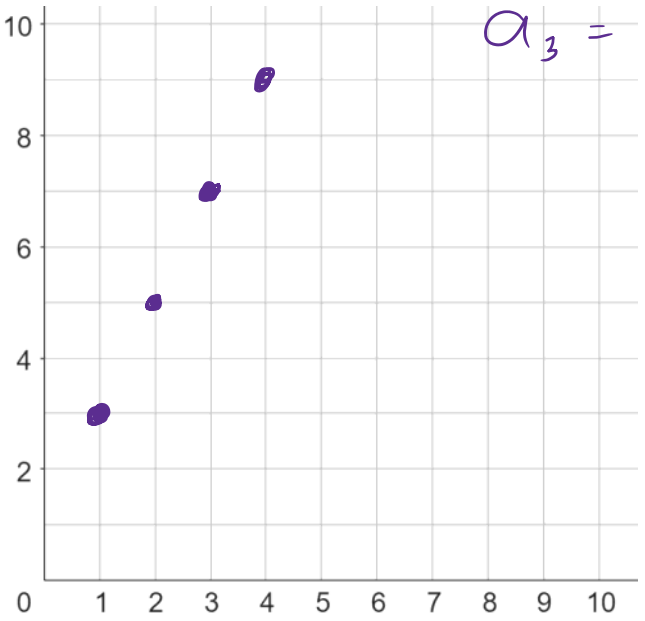



$a_n = 2n + 1$

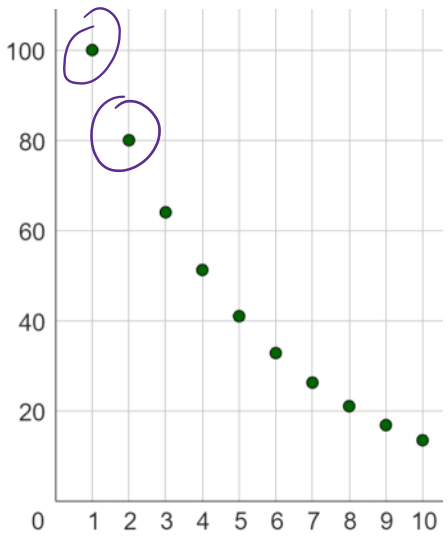
$a_1 = 3$

$a_2 = 5$

$a_3 = 7$



R) Find $g_{10}$ for the sequence graphed




$g_1 = 100$

$g_2 = 80$

$R = \frac{80}{100} = \frac{8}{10} = \frac{4}{5}$

$g_{10} = g_1 R^9$

$g_{10} = 100\left(\frac{4}{5}\right)^9$

$g_{10} = g_2 R^8$

$g_{10} = 80\left(\frac{4}{5}\right)^8$

S) Find the first term of an arithmetic sequence with…

$a_4 = 16, a_{10} = 46$

$a_{10} = a_4 + 6d$

$46 = 16 + 6d$

$30 = 6d$

$5 = d$

$a_1 = a_4 + d(-3)$

$a_1 = 16 + (5)(-3)$

$a_1 = 16 - 15$

$a_1 = 1$

$a_n = a_4 + d(n-4)$

$a_n = 16 + (5)(n-4)$

$a_1 = 16 + 5(-3) = 1$

T) Write a formula for the general term (nth term) of the geometric sequence. Then, find the 10th term.

$4, 8, 16, 32, \ldots$

U) Find the 10th number in the geometric sequence

$a_4 = 54, r = \frac{2}{3}$

V) Find the 7th term of the geometric sequence

$g_3 = \frac{16}{3}$ and $g_5 = \frac{64}{27}$

$g_5 = g_3 R^2$

$\frac{64}{27} = \frac{16}{3} R^2$

$\frac{\cancel{3}^{1}}{\cancel{16}_{4}} \cdot \frac{\cancel{64}^{4}}{\cancel{27}_{9}} = R^2$

$\frac{4}{9} = R^2$

$\pm \frac{2}{3} = R$

$g_7 = g_5 \cdot R^2$

$g_7 = \frac{64}{27} \cdot \frac{4}{9}$

$g_7 = \frac{256}{243}$

$24, \quad -12, \quad 6, \quad -3, \quad \ldots$

$R = -\frac{1}{2}$